

CS 206 - Introduction to Discrete Structures II

October 7, 2016

Homework: 4

Due Date: Friday, October 14 (2:00 pm)

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Assignment 1:

Suppose that a die is rolled twice. What are the possible values that the following random variables can take on:

1. the maximum value to appear in the two rolls;
2. the minimum value to appear in the two rolls;
3. the sum of the two rolls;
4. the value of the first roll minus the value of the second roll?

Moreover, calculate the probabilities associated with the random variables in parts (1) through (4).

Assignment 2:

Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let X denote the number of times player 1 is a winner. Find $\Pr[X = i]$ for $i = 0, 1, 2, 3, 4$.

Assignment 3:

Suppose that the distribution function of X is given by

1. $F(b) = 0$ for $b < 0$;
2. $F(b) = \frac{b}{4}$ for $0 \leq b < 1$;
3. $F(b) = \frac{1}{2} + \frac{b-1}{4}$ for $1 \leq b < 2$;
4. $F(b) = \frac{11}{12}$ for $2 \leq b < 3$;
5. $F(b) = 1$ for $3 \leq b$;

Find $\Pr[X = i]$ for $i = 1, 2, 3$, and also find $\Pr[\frac{1}{2} < X < \frac{3}{2}]$.

Assignment 4:

Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus.

1. Which of $\mathbf{E}x[X]$ or $\mathbf{E}x[Y]$ do you think is larger? Why?
2. Compute $\mathbf{E}x[X]$ and $\mathbf{E}x[Y]$.