CS 206 - Introduction to Discrete Structures II

September 16, 2016

Example: Set 1	Instructor: Morteza Monemizadeh
Due Date: :-)	TA: Hareesh Ravi

Note:

More examples on Induction and probability will be given on Wednesday, September 21.

Assignment 1:

What is the probability of the following events?

1. If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black balls, what is the probability that one of the balls is white and the other two black? Here, assuming that "randomly drawn" means that each outcome in the sample space is equally likely to occur.
2. A subgroup of 5 people are to be randomly selected from a group of 20 individuals consisting of 10 married couples. What is the probability that the 5 chosen are all unrelated? (That is, no two are married to each other.)

Solution: For the first part, we have $\Pr(E) = \frac{\binom{6}{1}\cdot\binom{5}{2}}{\binom{11}{3}}$. For the second part of the homework, let E be the event that the 5 chosen are all unrelated. We either have $\Pr(E) = \frac{\binom{10}{5}\cdot 2^5}{\binom{20}{5}}$ which is the same as $\Pr[E] = \frac{20\cdot 18\cdot 16\cdot 14\cdot 12}{20\cdot 19\cdot 18\cdot 17\cdot 16}$.

Assignment 2:

An urn contains n balls, one of which is special. If k of these balls are withdrawn one at a time, with each selection being equally likely to be any of the balls that remain at the time, what is the probability that the special ball is chosen?

Solution: Let E be the event that the special ball is chosen. We then have $\Pr(E) = \frac{\binom{1}{1}\cdot\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{k}{n}$.

Assignment 3:

A total of 36 members of a club play tennis, 28 play squash, and 18 play badminton. Furthermore, 22 of the members play both tennis and squash, 12 play both tennis and badminton, 9 play both squash and badminton, and 4 play all three sports. How many members of this club play at least one of three sports?

Solution: Let T be the set of members that plays tennis, S be the set that plays squash, and B be the set that plays badminton. Then, we have

$$\Pr[T \cup S \cup B] = \Pr[T] + \Pr[S] + \Pr[B] - \Pr[T \cap A] - \Pr[T \cap B] - \Pr[S \cap B] + \Pr[T \cap S \cap B] = \frac{43}{N}.$$

Assignment 4:

If n people are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year? How large need n be so that this probability is less than $\frac{1}{2}$?

Solution: $\frac{(365)\cdot(365-1)\cdots(365-n+1)}{(365)^n}$.

Assignment 5:

Compute the probability that 10 married couples are seated at random at a round table, then no wife sits next to her husband.

Solution: We explain the solution on Wednesday, September 21.
Hint: Use the fact that $\Pr[E_1 \cap E_2 \cdots E_n] = \frac{2^n (19-n)!}{(19)!}$ and the Inclusion-Exclusion identity.

Assignment 6:

A 5-card poker hand is said to be a full house if it consists of 3 cards of the same denomination and 2 other cards of the same denomination (of course, different from the first denomination). Thus, one kind of full house is three of a kind plus a pair. What is the probability that one is dealt a full house?

Hint: A "standard" deck of playing cards consists of 52 Cards in each of the 4 suits of Spades, Hearts, Diamonds, and Clubs. Each suit contains 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King.
Solution: We explain the solution on Wednesday, September 21.