ERRATA AND UPDATES FOR *AN INVITATION TO THE ROGERS–RAMANUJAN IDENTITIES*

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1. Errata

Many thanks to Michael D. Hirschhorn (University of New South Wales), Jeremy Lovejoy (Université Paris Diderot - Paris 7), Michael J. Schlosser (Universität Wien), Jim Lepowsky (Rutgers), and Koichi Takase for detecting and pointing out the errors corrected below.

- p. 20, Eq. (1.24) should be
$$a_0(n)f(n) + a_1(n)f(n+1) = 0.$$
- p. 27, line 2 should be
$$(\eta - 1)F(n,k) = (\kappa - 1)G(n,k).$$
- p. 44, Eq. (1.89),
$$\left(\binom{n}{j}\right)_q^3$$
should be
$$\left(\binom{n}{j}\right)_{q^3}.$$
- p. 45, Eq. (1.94), on the right side, z should be t.
- p. 55, line -10: Delete "Notice that we may rewrite $G(z)$ as a bilateral sum,
$$G(z) = \sum_{m=-\infty}^{\infty} \frac{(-1)^m q^{m(5m-1)/2}(z;q)_m}{(1-z)(q;q)_m},$$
and". Change the next word "we" to "We" and leave the sequel intact.
- p. 58, Eq. (2.19), right hand side numerator should be $f(-q^2, -q^7)$.
- p. 58, Eq. (2.20), right hand side numerator should be $f(-q, -q^8)$.
- p. 64, third displayed equation, the ordered pair on the right side should be reversed.
- p. 65, Eq. (2.48), right hand side numerator should be $f(-q^2, -q^3)$.
- p. 78, Eq. (2.75),
$$\left(\binom{n}{j}\right)_q^3$$
should be
$$\left(\binom{n}{j}\right)_{q^3}.$$
- p. 81, The sentence immediately preceding "*Proof of Theorem 2.42*" is misplaced, and should be deleted.
- p. 90, end of initial paragraph: "Rhodes" should be "Rhoades".

- p. 95, line 3, should be:
$$D(s) = O(|\Im s|^{C_1}).$$
- p. 108, Eq. (3.9), Left side numerator: exponent on q should be $n_1^2+n_2^2+n_1$.
- p. 109, Eqs. (3.12) and (3.13), numerator of multisum term: exponent on q should be $n_1^2 + n_2^2 + \cdots + n_{k-1}^2 + n_1 + n_2 + \cdots + n_{k-i}$.
- p. 111, Eq. (3.18) should read:
$$\sum_{n_k \geqq n_{k-1} \geqq \cdots \geqq n_1 \geqq 0} \frac{q^{n_1^2+n_2^2+\cdots+n_k^2}}{(q;q)_{n_k-n_{k-1}}(q;q)_{n_{k-1}-n_{k-2}}\cdots(q;q)_{n_2-n_1}(q;q)_{2n_1}} = \prod_{\substack{j>0\\ j\not\equiv 0,\pm(k+1) \pmod{6k+4}\\ j\not\equiv \pm(4k+2) \pmod{12k+8}}} \frac{1}{1-q^j},$$
cf. [And84, p. 269, Eq. (1.8)].
- p. 112, line 15, numerator of multisum term, exponent on q should be $n_1^2 + n_2^2 + \cdots + n_{k-1}^2 + n_1 + n_2 + \cdots + n_{k-i}$.
- p. 123, line 15: "is the *MacMahon–Schur*..." should be "as the *MacMahon–Schur* ...".
- p. 141, last line: remove the word "positive".
- pp. 150–151. In each of the first five Kanade–Russell conjectures presented, the last condition contains a typo. Specifically, in conjectures 5.8, 5.9, and 5.10, the last condition on λ should read:
$$\lambda_i - \lambda_{i+1} \leqq 1 \Longrightarrow \lambda_i + \lambda_{i+1} \equiv 0 \pmod 3.$$
The final condition on λ in Conjecture 5.12 should read:
$$\lambda_i - \lambda_{i+1} \leqq 1 \Longrightarrow \lambda_i + \lambda_{i+1} \equiv 2 \pmod 3,$$
and the final condition on λ in Conjecture 5.13 should read:
$$\lambda_i - \lambda_{i+1} \leqq 1 \Longrightarrow \lambda_i + \lambda_{i+1} \equiv 1 \pmod 3.$$
- p. 169, line −6 should read: $f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty$.
- p. 164, Eq. (6.19) right hand side should be
$$\frac{(-1)^m q^{-m(m-1)/2} E_{m-2}(q)}{(q,q^4;q^5)_\infty} - \frac{(-1)^m q^{-m(m-1)/2} D_{m-1}(q)}{(q^2,q^3;q^5)_\infty}.$$
- p. 165, Eq. (6.20), right hand side should be
$$(-1)^m q^{-m(m-1)/2}\big(E_{m-2}(q)D_\infty(q) - D_{m-1}(q)E_\infty(q)\big).$$
- p. 176, Eq. (A. 63): (**S. 48**−) should be (**S. 48**).
- p. 229, "Bringmann, Katherin" should be "Bringmann, Kathrin".

2. Updates

- On June 4, 2018, Michael Schlosser posted a preprint on the arxiv, (`https://arxiv.org/abs/1806.01153`) "Bilateral identities of the Rogers–Ramanujan type," which includes many beautiful bilateral RR-type identities, e.g.,

$$\sum_{k=-\infty}^{\infty} \frac{q^{k(5k-3)}}{(q;q^5)_k} = \frac{(q^4;q^5)_\infty f(-q^{10},-q^{15})}{(q^2;q^5)_\infty (q^3;q^5)_\infty},$$

$$\sum_{k=-\infty}^{\infty} \frac{q^{(k-1)(5k-1)}}{(q^2;q^5)_k} = \frac{(q^3;q^5)_\infty f(-q^{5},-q^{20})}{(q;q^5)_\infty (q^4;q^5)_\infty},$$

$$\sum_{k=-\infty}^{\infty} \frac{q^{k(5k-4)}}{(q^3;q^5)_k} = \frac{(q^2;q^5)_\infty f(-q^{5},-q^{20})}{(q;q^5)_\infty (q^4;q^5)_\infty},$$

$$\sum_{k=-\infty}^{\infty} \frac{q^{k(5k+3)}}{(q^4;q^5)_k} = \frac{(q;q^5)_\infty f(-q^{10},-q^{15})}{(q^2;q^5)_\infty (q^3;q^5)_\infty},$$

  where $f(a,b)$ is defined on p. 37, Eq. (1.59).
- §4.5, p. 135 ff. At *Combinatory Analysis 2018: A Conference in Honor of George Andrews' 80th Birthday*, held June 21–24, 2018, Chen Wang, a research fellow at the University of Vienna, announced a proof of the Borwein conjecture!
- §6.2, p. 159. At the time I wrote the book, I was unaware of the following paper: Michael D. Hirschhorn, A continued fraction of Ramanujan, *J. Austral. Math. Soc. Ser. A* **29** (1980) 80–86. In it, Hirschhorn gives a finite version of a very general continued fraction from Ramanujan's lost notebook, that includes the Gordon continued fraction and many others as special cases.
- §6.3, p. 159 ff. In an email to me dated July 5, 2018, Mike Hirschhorn shared that in about 1980, not long after the award of his Ph.D., he unexpectedly received a letter from Rodney Baxter asking how to prove six (or perhaps eight) Rogers–Ramanujan type identities. Mike recalls proving two of them, perhaps (A. 157) and (A. 158) from this book, in a return letter to Baxter, and suggesting he contact George Andrews about the rest. In an article by Baxter on the occasion of his 75th birthday in Cairns in 2015, he states that "I had helpful responses from Michael Hirschorn (sic), David Bressoud, Richard Askey and George Andrews."
- p. 165ff. This is both a correction and an update. In an email on April 16, 2019, Robert Osburn pointed out the following:

  > On page 165, you state that Garoufalidis and Le prove (6.21) and (6.22). This is not true.
  >
  > In the first version of [GL15], see Corollary 1.11, `https://arxiv.org/abs/1112.3905v1` they refer to the (still) unpublished paper [GZ11] for the proof. Note that Corollary 1.11 does not appear in the second version of [GL15], see `https://arxiv.org/abs/1112.3905v2` nor in the the final version of [GL15]. See the attached paper. [S. Garoufalidis and T. Lê, Nahm sums, stability and the

colored Jones polynomial, *Research in the Mathematical Sciences* **2**:1 (2015), 55 pp.]

It is better to state that Andrews provided a q-series proof for the three identities corresponding to the 3_1, 4_1 and 6_3 knots.

In the meantime, all of the conjectured q-series identities stated in Table 6 of the attached paper [Ibid.] have been proven. See
`http://maths.ucd.ie/~osburn/RRknots8.pdf` and
`http://maths.ucd.ie/~osburn/boqtails5.pdf` .

Version of October 11, 2022.